Ratios and Proportional Relationships (7.RP.A.1-3)

Analyze proportional relationships and use them to solve real-world and mathematical problems.

Finding the Unit Rate

A Ratio is a comparison of two numbers or measurements.

A Rate is a ratio in which the terms are in different units.

A Unit Rate is rate that is expressed as a quantity of ONE.

Examples:

Maggie bought 3.5 lbs of strawberries for $12.95. Find the cost per pound.

$$\frac{\text{cost}}{\text{pounds}} \quad \frac{\$12.95}{3.5} = \frac{n}{1}$$

$$\frac{3.5n}{3.5} = \frac{12.95}{3.5}$$

$$n = \$3.70$$

$3.70 per pound

1. Multiply the diagonals.
2. Divide to solve for the variable.

Joey walks 1/2 of a mile in 1/8 of an hour. How far does he walk per hour?

$$\frac{1/2}{1/8} = \frac{n}{1} \quad \frac{\text{miles}}{\text{hours}}$$

$$\tfrac{1}{8}n = \tfrac{1}{2}$$

$$n = 4$$

4 miles per hour

How to Determine if a Relationship is Proportional

From 2 Ratios:

CROSS MULTIPLY! If the products are equal, then it's proportional.

$$\frac{12}{17} = \frac{36}{51}$$

$(12 \cdot 51) = 17 \cdot 36$

612 = 612

From a Table:

Find each unit rate. If they are all the same, then it's proportional!

Tickets Bought	Cost $
6	$210
11	$385
25	$875

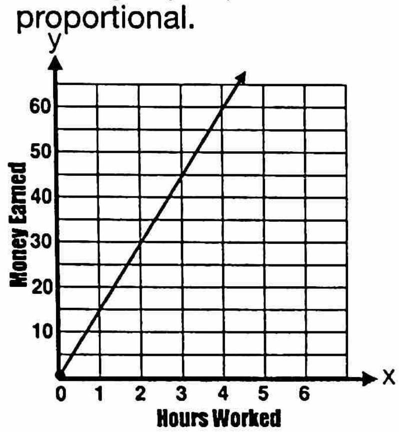
$210 ÷ 6 = **$35**

$385 ÷ 11 = **$35**

$875 ÷ 25 = **$35**

From a Graph:

If the graph is a straight line that passes through the origin (0, 0), then it's proportional.



Constant of Proportionality

The constant of proportionality is simply the unit rate. If you earn $16 for working 2 hours, then the constant of proportionality is $8 because that's the amount you earn for one hour. ($16 ÷ 2 = $8)

From a Table

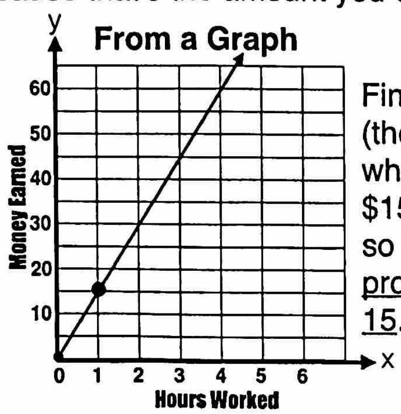
Month	Money Saved
3	$150
12	$600

Find the unit rate.

$150 ÷ 3 = **$50** ← Constant of proportionality.

$600 ÷ 12 = **$50**

From a Graph



Find the unit rate (the value of y when $x = 1$). $15 for one hour, so the constant of proportionality is 15.

From an Equation

$$y = 15x$$

The constant of proportionality is the number that is multiplied to "x".

Writing Equations for Proportional Relationships ($y = kx$)

When relationships are proportional, you can write an equation in the form $y = kx$. The constant of proportionality (k) is the unit rate. It is the number multiplied by the input (x) to determine the output (y).

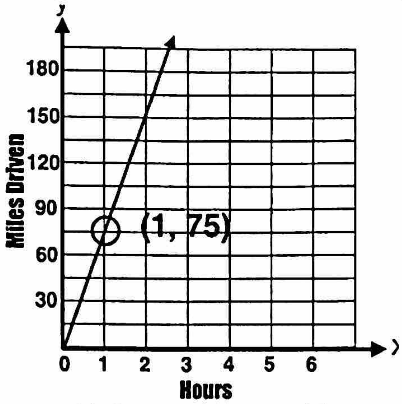
Find the constant of proportionality, k, and replace that value in the equation $y = kx$.

The total cost is proportional to the number of t-shirts purchased.

Kellie bought 19 t-shirts and spent $242.25. Write an equation, in the form $y = kx$, to represent this relationship.

$242.25 ÷ 19 = $12.75

y = 12.75x



Unit rate is 75 mi/h

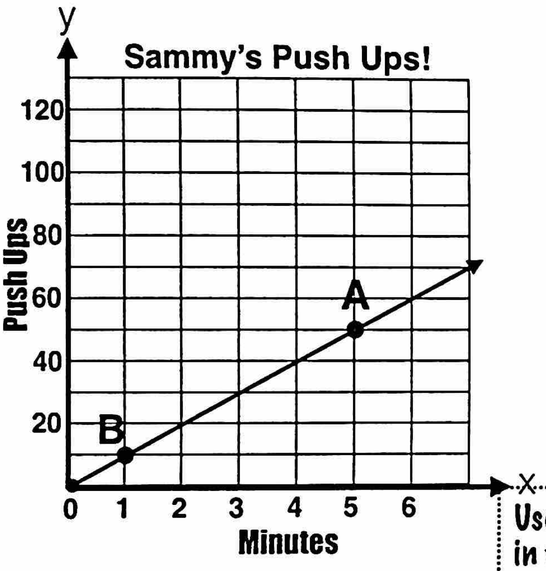
y = 75x

Bags	# of Jolly Ranchers
2	126
5	315
6	378

126 ÷ 2 = 63
315 ÷ 5 = 63
378 ÷ 6 = 63

y = 63x

What's Does an Ordered Pair (x, y) Mean? (Proportional Relationships in Context)



Point A (5,50) means that in 5 minutes, Sammy did 50 push ups. The x-value (5) refers to the number of minutes and the y-value (50) represents the number of push-ups.

Point B (1,10) represents the unit rate or the constant of proportionality. It means that in 1 minute, Sammy did 10 push-ups.

Whenever you need to find the unit rate (constant of proportionality, k), go to x = 1 on the graph and then move up to find the value of y. This will always give you the amount for one.

Use the labels from the graph to describe the numbers in the ordered pair. That's how you put it in context!

Using Proportional Relationships to Solve Multi-Step Ratio and Percent Problems

You can use proportions to solve problems involving ratio and percent.

Ratios

Linda sold 49 boxes of cookies in 3 days. If the rate remains the same, how many boxes of cookies will she sell in 18 days?

$$\frac{\textit{cookies}}{\textit{days}} \quad \frac{49}{3} = \frac{n}{18}$$

$$\frac{3n}{3} = \frac{882}{3}$$

n = 294

Percent Problems

$$\boxed{\frac{part(is)}{whole(of)} = \frac{\%}{100}}$$ ←Use this ratio!

What is 5% of 118? $\frac{n}{118} = \frac{5}{100}$ 100n = 590 **n = 5.9**

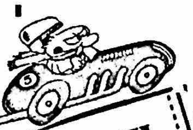
24 is what percent of 60? $\frac{24}{60} = \frac{n}{100}$ 60n = 2400 **n = 40%**

64 is 40% of what number? $\frac{64}{n} = \frac{40}{100}$ 40n = 6400 **n = 160**

__Discount__ - an amount that you are saving. Subtract it from the original price.
__Sales Tax__ - an amount that you are paying. Add it to the original price.

The Number System (7.NS.A.1)

Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers.



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Operations with Integers

	Addition	Subtraction	Multiplication	Division
Same Signs	Add the numbers. Keep the sign.	"Add the opposite" and then use the rules of addition.	When the signs are the same, the product is positive.	When the signs are the same, the quotient is positive.
	7 + 8 = 15 -5 + (-4) = -9	5 - 11 5 **+ (-11)** = -6	5 • 4 = 20 -3 • (-8) = 24	16 ÷ 2 = 8 -18 ÷ -3 = 6
Different Signs	Subtract the numbers. Keep the sign of the number with the larger absolute value.	"Add the opposite" and then use the rules of addition.	When the signs are different, the product is negative.	When the signs are different, the quotient is negative.
	20 + (-7) = 13 6 + (-10) = -4	-9 - 11 -9 **+ (-11)** = -20	12 • (-2) = -24 -10 • 10 = -100	-60 ÷ 12 = -5 56 ÷ -7 = -8

- The Absolute Value of a number is its distance from zero on the number line. (always positive)
- The Additive Inverse is the opposite of a number.
- The Additive Inverse Property states that any number plus its opposite equals zero. a + (-a) = 0

Properties of Addition and Multiplication

Commutative Property *The order of the numbers can change.*	a + b = b + a ab = ba	5 + 9 = 9 + 5 3 • 8 = 8 • 3
Associative Property *The numbers can be grouped differently but the order stays the same.*	(a + b) + c = a + (b + c) (a • b) • c = a • (b • c)	(4 + 3) + 7 = 4 + (3 + 7) (6 • 2) • 5 = 6 • (2 • 5)
Distributive Property *Multiply the number outside the parentheses to each number inside.*	a(b + c) = ab + ac a(b - c) = ab - ac	5(10 + 2) = 5(10) + 2(10) 3(9 - 1) = 3(9) - 3(1)
Identity Property *The sum or product is the same number you started with.*	a + 0 = 1 a • 1 = a	57 + 0 = 57 25 • 1 = 25
Additive Inverse Property *The sum of any number and its opposite is zero.*	a + (-a) = 0	16 + (-16) = 0
Zero Property of Multiplication *The product of any number and zero is zero.*	a • 0 = 0	21 • 0 = 0

The Number System (7.NS.A.2-3)

Apply and extend previous understandings of multiplication and division of fractions and to multiply and divide rational numbers. Solve real-world and mathematical problems involving the four operations with rational numbers.

Order of Operations

P	Parentheses
E	Exponents
M/D	Multiply or Divide (from left to right)
A/S	Add or Subtract (from left to right)

$$-5(-4)-2\left[\frac{4}{3}(6)-5\right]+6^2\div 9$$
$$-5(-4)-2[8-5]+6^2\div 9$$
$$-5(-4)-2[3]+6^2\div 9$$
$$-5(-4)-2[3]+36\div 9$$
$$20 - 6 + 4$$
$$14 + 4$$
$$18$$

Operations with Fractions

Addition & Subtraction Add or subtract the numerators, keep the denominator.	*If the denominators are not the same, find a common denominator, write equivalent fractions, and then add or subtract. $\frac{5}{9}+\frac{2}{9}=\frac{7}{9}$ $3\frac{4}{5}-\frac{1}{2}$ $\frac{19}{5}-\frac{1}{2}$ $\frac{38}{10}-\frac{5}{10}=\frac{33}{10}=3\frac{3}{10}$
Multiplication Multiply the numerators. Multiply the denominators. Simplify.	*Change mixed numbers to improper fractions. $\frac{5}{6}\bullet\frac{2}{5}=\frac{10}{30}=\frac{1}{3}$ $2\frac{1}{2}\bullet\frac{3}{4}$ $\frac{5}{2}\bullet\frac{3}{4}=\frac{15}{8}=1\frac{7}{8}$
Division Multiply the first fraction by the <u>reciprocal</u> of the second fraction.	*Change mixed numbers to improper fractions. $\frac{4}{5}\div\frac{8}{9}$ $\frac{4}{5}\bullet\frac{9}{8}=\frac{36}{40}=\frac{9}{10}$ $3\frac{1}{5}\div 2\frac{2}{3}$ $\frac{16}{5}\div\frac{8}{3}=\frac{16}{5}\bullet\frac{3}{8}=\frac{48}{40}=1\frac{8}{40}=1\frac{1}{5}$

How Can I Tell if the Product will be Positive or Negative?

If there are an EVEN number of negative factors, the product is POSITIVE.

$$(-5)(-2)=10$$

2 Factors = Positive

$$\left(-\frac{1}{2}\right)\left(-\frac{1}{3}\right)\left(-\frac{1}{2}\right)\left(-\frac{2}{5}\right)=\frac{2}{60}$$

4 Factors = Positive

If there are an ODD number of negative factors, the product is NEGATIVE.

$$(-10)(-2)(-5)(-10)(-1)=-1000$$

5 Factors = Negative

Expressions & Equations 7.EE.A.1-2

Use properties of operations to generate equivalent expressions.

Using the Distributive Property

To simplify expressions using the distributive property, multiply the number (or variable) outside the parentheses to each number (or variable) inside the parentheses. Watch signs!

$4(x + 5)$	$-3(x - 6)$	$\frac{1}{2}\left(2x+\frac{4}{5}\right)$
$4(x) + 4(5)$	$-3(x) + -3(-6)$	$\frac{1}{2}(2x)+\frac{1}{2}\left(\frac{4}{5}\right)$
$\mathbf{4x + 20}$	$\mathbf{3x + 18}$	$x+\frac{2}{5}$

Combining Like Terms

"Like terms" are terms that have the same exact variables (and exponents). To combine "like" terms, add the coefficients (use integer rules) and keep the variables.

$4x + 9x$	$-7n + 12 - 4n$	$5a + b - 10a + 7b$	$3x^2 - 7x + 9x - 5x^2$
$4x + 9x$	$-7n + 12 - 4n$	$5a \boxed{+ b} - 10a \boxed{+ 7b}$	$3x^2 \boxed{- 7x} \boxed{+9x} - 5x^2$
$\mathbf{13x}$	$\mathbf{-11n + 12}$	$\mathbf{-5a + 8b}$	$\mathbf{-2x^2 + 2x}$

Common Factors

A common factor is a number that goes into two or more given numbers evenly.
The greatest common factor is the <u>largest</u> number that goes into two or more given numbers evenly.
Include the *highest possible* exponent of each variable that they *ALL* have. Do variables separately.

	24, 56	$18x^2, 27x^3$	$12xy^2z + 8x^2y^2 - 20x^3y^2z$
Factors	<u>24</u>: 1, 2, 3, 4, 6, 8, 12, 24 <u>56</u>: 1, 2, 4, 7, 8, 14, 28, 56	<u>18</u>: 1, 2, 3, 6, 9, 18 <u>27</u>: 1, 3, 9, 27	<u>12</u>: 1, 2, 3, 4, 6, 12 <u>8</u>: 1, 2, 4, 8 <u>20</u>: 1, 2, 4, 5, 10, 20
Common Factors	2, 4, 8	3, 9	2, 4
Greatest Common Factor	8	$9x^2$ * Since they both have <u>at least</u> x^2, include it in the GCF.	$4xy^2$ * You cannot include "z" because the second term has no "z".

Equivalent Expressions with Percent

Write an expression to represent the following situation: Liz buys an iPad, *p*, plus 7% tax.

*Covert 7% to a decimal by moving the decimal 2 places to the left. 7% = .07

You can write the expression 2 different ways:

$p + .07p$ This represents the price of the iPad (p), plus 7% of the price ($.07p$).

$1.07p$ This represents the total price. Just add the like terms together in the first expression.

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Expressions & Equations 7.EE.B.3-4

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

Rules for Solving Equations

- Whatever is done to one side must also be done to the other side.
- Use inverse operations to isolate the variable. (Get it alone on one side of the equals sign.)
- Get rid of the parentheses first! (Use the distributive property.)

1. Use the distributive property. $4(2x + 7) = 108$
2. Subtract 28 from each side (inverse of addition).

$$8x + \cancel{28} = 108$$
$$\underline{-\cancel{28} \quad -28}$$

3. Divide each side by 8 to isolate the variable (x).

$$\frac{\cancel{8}x}{\cancel{8}} = \frac{80}{8}$$
$$x = 10$$

Solving Real-World Multi-Step Equations

- Define the variable. (I like to use a variable that "makes sense" in the problem.)
- Write the equation.
- Use inverse operations to solve for the variable.

Aubree is saving for a new pair of Uggs that cost \$250. So far, she has \$70. If she saves \$45 each week, how many weeks will it take her to have enough money to buy the Uggs?

1. Define a variable. Let w = week (makes sense!)
2. Write the equation. (\$70 to start + \$45 per week) $\cancel{70} + 45w = 250$
3. Use inverse operations to solve for "w". $\underline{-\cancel{70} \qquad -70}$

$$\frac{\cancel{45}w}{\cancel{45}} = \frac{180}{45}$$

Aubree can buy the Uggs in 4 weeks! $w = 4$

Solving Real-World Multi-Step Equations with Percent

Mark earns \$200 each week plus 2.5% of his sales. How much does he need in sales to earn \$287.50 in one week?

1. Define a variable. Let s = sales (makes sense!)
2. Write the equation. (\$200 to start + 2.5% of his sales) $\cancel{200} + .025s = 287.50$
3. Use inverse operations to solve for "w". $\underline{-\cancel{200} \qquad -200}$

$$\frac{\cancel{.025}s}{\cancel{.025}} = \frac{87.50}{.025}$$

Mark needs \$3500 in sales to earn \$287.50. $s = 3500$

Geometry 7.G.A.1-3

Draw construct, and describe geometrical figures and describe the relationships between them.

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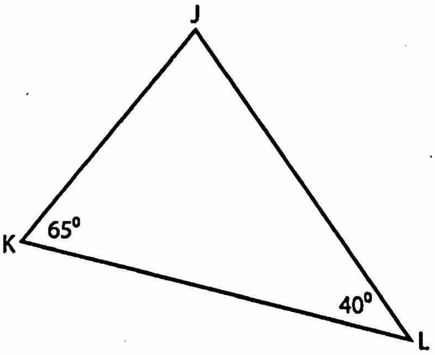
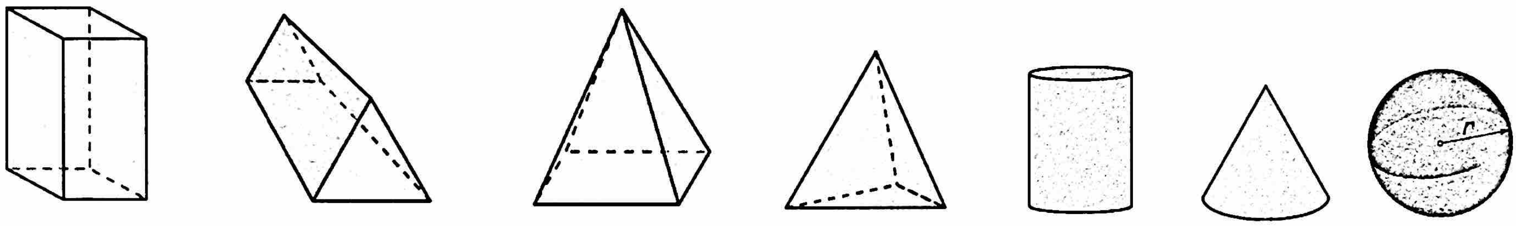
Triangles

The sum of the angle measures in any triangle is 180º.



$$65°+40°+\angle J = 180°$$
$$105°+\angle J = 180°$$
$$-105° \qquad -105°$$
$$\angle J = 75°$$

3-Dimensional Figures

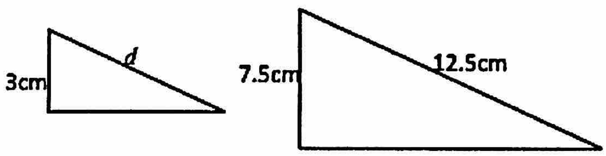


Rectangular Prism | Triangular Prism | Rectangular Pyramid | Triangular Pyramid | Cylinder | Cone | Sphere

Scale Drawings/Similar Figures

<u>Similar Figures</u> - Figures that are the same shape, but not necessarily the same size.
<u>Corresponding Sides</u> - Sides in similar figures whose lengths are proportional.
<u>Corresponding Angles</u> - Always congruent with figures are similar.
<u>Scale Factor</u> - Ratio of two corresponding lengths in two similar geometric figures.

To find a missing side length, set up a proportion and solve!



$$\frac{3}{7.5} = \frac{d}{12.5}$$ <u>smaller figure</u> / larger figure

$$\frac{7.5d}{7.5} = \frac{37.5}{7.5}$$

$$d = 5$$

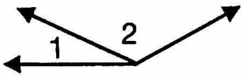
1. Multiply the diagonals.
2. Divide to solve for the variable.

Geometry 7.G.B.4-6

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

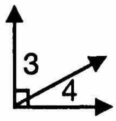
Angle Pairs

Adjacent Angles



2 angles that are next to each other.

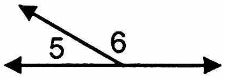
Complementary Angles



2 angles whose sum is 90º.

If angle 3 = 60º, then angle 4 = 30º.
60º + 30º = 90º

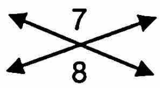
Supplementary Angles



2 angles whose sum is 180º

If angle 5 = 42º, then angle 6 = 138º.
42º + 138º =180º

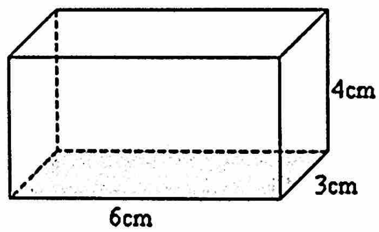
Vertical Angles



2 angles that are opposite each other.

If angle 7 = 136º, then angle 8 = 136º.
because they are equal to each other.

Surface Area and Volume of a Rectangular Prism



Surface Area - The sum of the areas of all 6 sides.

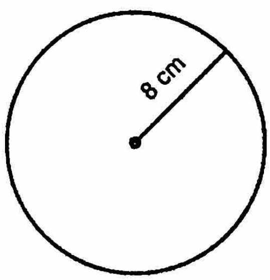
Top:	$6 \times 3 = 18$ cm^2
Bottom:	$6 \times 3 = 18$ cm^2
Front:	$6 \times 4 = 24$ cm^2
Back:	$6 \times 4 = 24$ cm^2
Side:	$3 \times 4 = 12$ cm^2
Side:	$3 \times 4 = 12$ cm^2
TOTAL:	**108 cm^2**

Volume: $l \cdot w \cdot h$

$6 \cdot 3 \cdot 4$

72 cm^3

Circumference and Area of a Circle



Circumference

$C = \pi \cdot d$

$C = 3.14 \cdot 16$

$C = 50.24$ cm

Area

$A = \pi \cdot r^2$

$A = 3.14 \cdot (8)^2$

$A = 200.96$ cm^2

Statistics and Probability 7.SP.A.1-4

Use random sampling to draw inferences about population.
Draw informal comparative inferences about two populations.

Measures of Central Tendency

Range: The difference between the lowest and highest numbers in a set of data.

Mode: The number that is repeated most often in a set of data.

Median: When ordered, the number (or average of 2 numbers) in the middle.

Mean: The average.

Gayle's Quiz Scores: 94, 78, 60, 80, 92, 94

First, write the scores in order from lowest to highest: 60, 78, 80, 92, 94, 94

Range: 94 - 60 = 34

Mode: 94

Median: (80 + 92) ÷ 2 = 86

Mean: (60 + 78 + 80 + 92 + 94 + 94) ÷ 6 = 83

Random Samples

A random sample consists of "n" objects, which are all equally likely to occur.
The chosen sample is representative of the population, so the conclusions will be valid.

Examples:

- Rolling a number cube
- Flipping a coin
- Spinning a numbered spinner
- Pulling pieces of paper out of a hat

Using Data from a Sample to Make a Prediction

You can use the information from a sample to make predictions by setting up a proportion.

Example:

The school cafeteria wants to add something new to their menu, so they randomly selected 60 students and asked them what they'd like added to the lunch menu. Twenty-eight of the students surveyed said they want Crab Fries. If there are 1080 students in the school, how many would you expect to choose Crab Fries?

$$\frac{28}{60} = \frac{n}{1080}$$

$$\frac{60n}{60} = \frac{30,240}{60}$$

$$n = 504$$

Text

Statistics and Probability 7.SP.C.5-8

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Investigate chance processes and develop, use, and evaluate probability models.

Probability

Probability is the likelihood of an event; it can be either theoretical or experimental.

Theoretical Probability: The ratio of ways the event can occur to the number of possible outcomes.

$$P(A) = \frac{\text{favorable outcomes}}{\text{possible outcomes}}$$

Experimental Probability: The ratio of the total number of times the favorable outcome happens to the total number of trials, or times, the experiment was performed.

$$P(A) = \frac{\text{favorable outcomes}}{\text{total number of trials}}$$

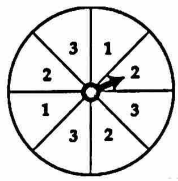
Roll a die.

P(5) = 1/6 (16.7%)

P(odd #) = 3/6 = 1/2 (50%)

P(7) = 0 (Impossible!)

Spin a spinner.



P(2) = 3/8 (37.5%)

P(# less than 4) = 8/8 = 1 (100%)

P(2 or 3) = 6/8 = 3/4 (75%)

Compound Events

An event that consists of two or more simple events whose outcomes are equally likely.

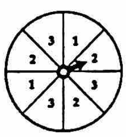
Independent Events

Events in which the first event does NOT affect the second event. They have nothing to do with each other! The number of possible outcomes will always remain the same.

Roll a die twice. Find the P(5, even #).

$$\frac{1}{6} \bullet \frac{3}{6} = \frac{3}{36} = \frac{1}{12}$$

Spin the spinner and flip a coin. Find the P(1, tails).



$$\frac{2}{8} \bullet \frac{1}{2} = \frac{2}{16} = \frac{1}{8}$$

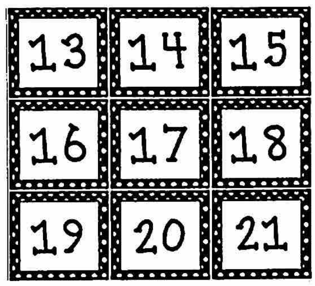
Put 10 cards numbered 0 - 9 in a hat. Choose a card, replace it, and choose a second card. Find P(7, # greater than 7).

$$\frac{1}{10} \bullet \frac{2}{10} = \frac{2}{100} = \frac{1}{50}$$

Dependent Events

Events in which the first event DOES affect the second event. The number of possible outcomes will change as the result of the first event.

Choose a card, do NOT replace it, and choose a second card.



Find P(16, 14). $\frac{1}{9} \bullet \frac{1}{8} = \frac{1}{72}$

Find P(even #, even #). $\frac{4}{9} \bullet \frac{3}{8} = \frac{12}{72} = \frac{1}{6}$